Microscopics of Extremal Kerr from Spinning M5 Branes

Amitabh Virmani

Université Libre de Bruxelles, Belgium

Indian Strings Meeting 2011, January 04 2011

Compere, de Buyl, Stotyn: 1006.5464 and work with Compere, Song: 1010.0685

Closely Related Work

The **five-dimensional** extremal black hole rotating in a single plane has been studied in

"Microscopic Realization of the Kerr/CFT Correspondence", Guica and Strominger, arXiv: 1009.5039

We describe the **four-dimensional** extremal rotating black hole.

Main claim

In an M-theory embedding of the extremal Kerr black hole, one can identify a dual theory in the near-horizon region given by

A specific deformation of the Discrete Light Cone Quantized (DLCQ) Maldacena-Strominger-Witten (MSW) CFT.

The Kerr-CFT conjectures that this deformed theory is a CFT

Our construction is consistent with the Kerr-CFT conjecture, it passes one non-trivial check.

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)

Outline

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$
4. Microscopics
   - DLCQ MSW CFT, deformation, Kerr-CFT

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$
4. Microscopics
   - DLCQ MSW CFT, deformation, Kerr-CFT

Five-dimensional minimal supergravity

Consider the following consistent truncation of M-theory on T^6

$$\begin{aligned}
\mathcal{L}_{11} &= R_{11} \star_{11} \mathbf{1} - \frac{1}{2} \star_{11} \mathcal{F} \wedge \mathcal{F} + \frac{1}{6} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{A}. \\
ds_{11}^2 &= ds_5^2 + dx_1^2 + dx_2^2 + \cdots dx_6^2 \\
\mathcal{A} &= \frac{1}{\sqrt{3}} A \wedge dx_1 \wedge dx_2 + \frac{1}{\sqrt{3}} A \wedge dx_3 \wedge dx_4 + \frac{1}{\sqrt{3}} A \wedge dx_5 \wedge dx_6
\end{aligned}$$

The resulting theory is five-dimensional minimal supergravity.

Five-dimensional minimal supergravity

- Simple yet very rich theory: supersymmetric solutions classified

$$\mathcal{L}_5 = R \star 1 - \frac{1}{2} \star F_{(2)} \wedge F_{(2)} + \frac{1}{3\sqrt{3}} F_{(2)} \wedge F_{(2)} \wedge A_{(1)}$$

- Consistent truncation by setting

$$A^1 = A^2 = A^3 = \frac{1}{\sqrt{3}} A \quad \text{in } \mathrm{U}(1)^3 \text{ 5d supergravity}$$

Can also be obtained from type IIB theory (M2-M2-M2 $\leftrightarrow$ D1-D5-P)

- Harbors supersymmetric rotating black holes

BMPV black hole   Supersymmetric black ring

Many interesting solutions of this theory are not known, e.g.,

Black ring describing thermal excitations over the susy black ring is not known.

Microscopics of certain black holes only partially understood.

Microscopics of magnetic black string, however, is reasonably well understood, in terms of the MSW CFT. So we add M5 charges on Kerr-string.

Many interesting solutions of this theory are not known, e.g.,

Black ring describing thermal excitations over the susy black ring is not known.

Microscopics of certain black holes only partially understood.

Microscopics of magnetic black string, however, is reasonably well understood, in terms of the MSW CFT. So we add M5 charges on Kerr-string.

To do this, we use techniques drawn from the supergravity literature, in particular, the use of hidden symmetries.

Dimensional Reduction to 3d

- Consider spacetimes with at least two commuting Killing vectors
- (Schematical) ansatz:

$$g^{(5)}_{\mu\nu} = \left(\begin{array}{c|c|c} g_{mn} & A_m & B_m \\ \hline A_n & \phi_2 & \chi_1 \\ \hline B_n & \chi_1 & \phi_1 \end{array}\right)$$

$$A^{(5)}_{\mu} = \left(\begin{array}{c|c|c} C_m & \chi_2 & \chi_3 \end{array}\right)$$

- D=3: a metric, 3 one-forms, and 5 scalars
- One-forms can be dualized into scalars

- After dualization of one-forms into scalars → 3d metric + 8 scalars
- If the reduction is done over one spacelike and one timelike direction

$$3\text{d} = \text{Euclidean gravity} + \frac{G_{2(2)}}{SO(2,2)}$$

- If the reduction is done over both spacelike directions

$$3\text{d} = \text{Lorentzian gravity} + \frac{G_{2(2)}}{SO(4)}$$

- $SO(4)$ is the maximal compact subgroup of $G_{2(2)}$: $\frac{G_{2(2)}}{SO(4)}$ is a Riemannian coset, whereas $\frac{G_{2(2)}}{SO(2,2)}$ is pseudo-Riemannian

- Riemannian cosets have played a key role in proofs of celebrated black hole uniqueness theorems
- Psuedo-Riemannian cosets have attracted much recent interest
  - Bossard, Nicolai, Stelle: stratified structure of BPS black holes
  - Bossard, Michel, Pioline: construction of 'fake superpotential'

In this work we use the pseudo-Riemannian $\frac{G_{2(2)}}{SO(2,2)}$ coset as a solution generating technique

This is achieved as follows

Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows

Solution S_5

Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows

Solution S_5

↓ *Reduction to 3d*

Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows

Solution S_5

↓ *Reduction to 3d*

Solution S_3

Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows

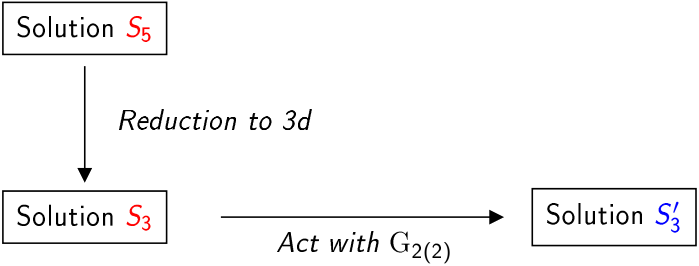
Solution S_5

↓ *Reduction to 3d*

Solution S_3 → *Act with* $\mathrm{G}_{2(2)}$

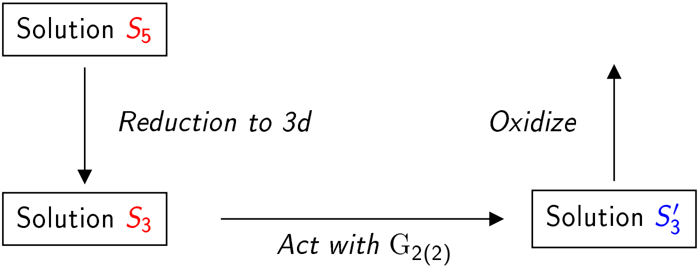
Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows



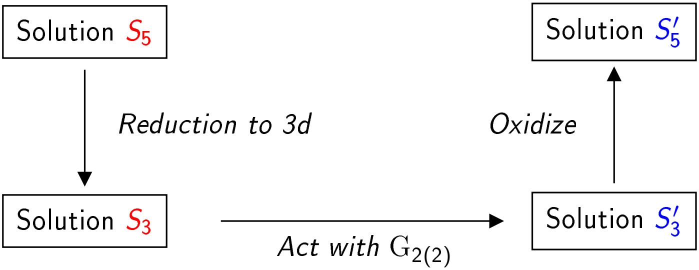
Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows



Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

This is achieved as follows



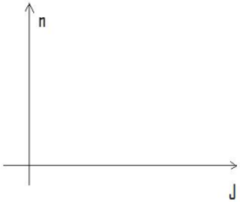
Gaiotto, Li, Padi: 2007, Bouchareb, Clement, et al: 2008
Berkooz, Pioline: 2008, Compère, de Buyl, Jamsin, A.V.: 2009

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$
4. Microscopics
   - DLCQ MSW CFT, deformation, Kerr-CFT

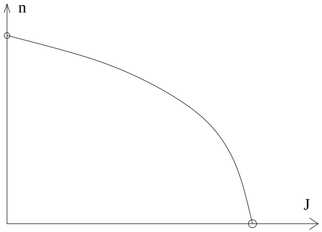
The Idea

Have a solution in which we can vary J_ϕ and magnetic charge independently, while maintaining extremality.



The Idea

Have a solution in which we can vary J_ϕ and magnetic charge independently, while maintaining extremality.



Use microscopic theory of the M5 branes; turn off the M5 charge while increasing J_ϕ.

Magnetic Black String: the construction

- Let us first reduce the theory over the spacelike string direction. We get

N=2, D=4 S^3 Supergravity (S=T=U in the 4d STU model)

- The problem now reduces to constructing an appropriate 4d asymptotically flat black hole in this theory.

Magnetic Black String: the construction II

- Theorem [Brietenlohner-Gibbons-Maison, 1986]:

All single-center spinning non-extremal black holes of this theory lie in the single SO(2,2) orbit that contains the Kerr black hole.

- Therefore,

the problem = finding the appropriate SO(2,2) element inside G2(2)

With a little group theory a construction is indeed possible.

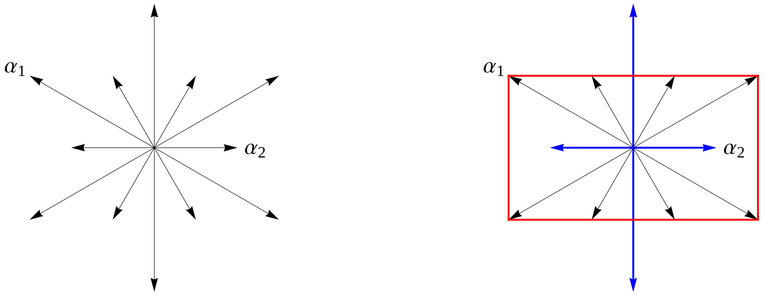


Figure: Root diagram of $\mathfrak{g}_{2(2)}$.

This problem is systematically solved by understanding which generator corresponds to which charge.

The Magnetic Black String

The magnetic black string solution carries three parameters

- Magnetic 1-brane charge (M5 charges)
- Rotation in the transverse space
- Energy above the BPS bound

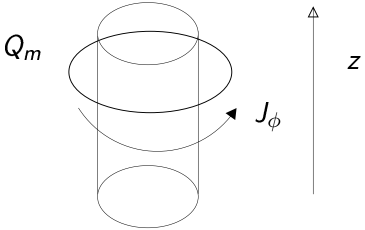
The Magnetic Black String

The magnetic black string solution carries three parameters

- Magnetic 1-brane charge (M5 charges)
- Rotation in the transverse space
- Energy above the BPS bound

	t	x_1	x_2	x_3	x_4	x_5	x_6	z	r	θ	ϕ
M5	×	—	—	×	×	×	×	×			
M5	×	×	×	—	—	×	×	×			
M5	×	×	×	×	×	—	—	×			

Physical properties of the spinning spinning one-brane



The black string has three independent charges

$$Q_m, \qquad J_\phi \qquad M$$

The M5 brane charge is a one-brane charge

$$\frac{\sqrt{3}}{2} n\, l_p = Q_m = \frac{1}{4\pi}\int_S F$$

We expect $n \in \mathbb{Z}$, $J_\phi \in \mathbb{Z}/2$. At extremality $T_H = 0$, $M(n, J_\phi)$ and the entropy is

$$S = 2\pi J_\phi$$

The entropy is the one of Kerr. The quartic invariant vanishes. No contribution from n.

To study thermodynamics of this solution, one needs to use the Copsey-Horowitz formalism.

There are non-zero angular **and linear** velocities Ω_ϕ and v_z!

The orientation of the linear velocity v_z is independent of J_ϕ but depends on the orientation of Q_m.

There is an interesting kinematics at the black hole horizon.

Standard decoupling limit [Maldacena, 1998]

Express the solution in terms of (n, J_ϕ)

$$\textit{Send} \qquad l_p \to 0, \qquad r \to r\, l_p^3$$

This is a decoupling limit involving a near-horizon limit. The resulting geometry is

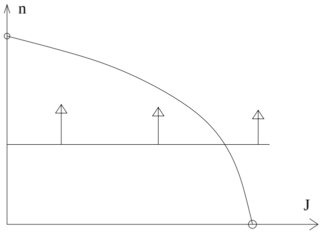
$$\textit{extremal } BTZ \times S^2$$

Brown-Henneaux central charge and levels :

$$c_L = c_R = \frac{3l}{2G_3} = 6n^3, \qquad h_L = \frac{2J_\phi^2}{n^3}, \qquad h_R = 0$$

[Larsen, '98]
[Compere, de Buyl, Stotyn, A.V., '10]

Limitation of the Maldacena limit

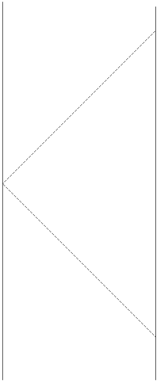


There is no supergravity description for n small. We need another route to explain the microscopics of Kerr where we turn $n \to 0$.

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$
4. Microscopics
   - DLCQ MSW CFT, deformation, Kerr-CFT

Near-horizon region as a decoupling limit



Now, (i) go to the co-moving frame and (ii) zoom on the horizon

$$r \to r_+ + \lambda r, \qquad t \to \frac{t}{\lambda}, \qquad \lambda \to 0\,.$$

Decoupling between horizon and asymptotics

This spacetime is geodesically complete.

$$ds^2 = S^1 \otimes_W AdS_2 \otimes_W S^2$$

Enhancement to $U(1)_z \times SL(2,\mathbb{R})_t \times U(1)_\phi$ symmetry [Kunduri, Lucietti, Reall, '07]

Near-horizon region: Explicit solution

We introduce the variables $R > 0$ and $\Phi \in [0, \frac{\pi}{2}]$ defined by

$$n = R\cos\Phi, \qquad a(J_\phi, n) = R\sin\Phi\,.$$

Then, an overall scale R^2 factors out,

$$\begin{aligned}\frac{ds^2}{R^2\, l_p^2} &= \Gamma(\theta)\left[-(k_\phi)^2 r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2\right] \\ &\quad + \gamma_{\phi\phi}(\theta) e_\phi^2 + 2\gamma_{\phi z}(\theta) e_\phi\, e_z + \gamma_{zz}(\theta) e_z^2 \\ \frac{A}{R\, l_p} &= f_\phi(\theta) e_\phi + f_z(\theta) e_z\ ,\end{aligned}$$

where $e_\phi = d\phi + k_\phi r dt$, $e_z = dz + k_z r dt$.

The functions only depend on $\Phi \in [0, \frac{\pi}{2}]$. What happens at $\Phi = 0$, $\Phi = \frac{\pi}{2}$?

Interpolating geometry at one end

$\Phi = \frac{\pi}{2}$. In this case, there are no M5^3 brane
$\Leftrightarrow n = 0, \qquad R^2 = \frac{4G_4 J_\phi}{l_p^2}$

The solution becomes the near-horizon decoupled region of the extremal Kerr black hole ($NHEK \times S^1$)

$$\frac{ds^2}{4G_4 J_\phi} = \Gamma(\theta)\left(\frac{dr^2}{r^2} + d\theta^2 - r^2 dt^2\right) + \gamma_{\phi\phi}(\theta)(d\phi + r dt)^2 + dz^2,$$

and $A = 0$ where

$$\Gamma(\theta) = \frac{1}{4}(1+\cos^2\theta),\ \gamma_{\phi\phi}(\theta) = \frac{\sin^2\theta}{1+\cos^2\theta}\,.$$

This is expected since the original extremal spinning one-brane solution reduces to Kerr $\times S^1$.

Interpolating geometry at the other end

$\Phi = 0$ In that case, there is no angular momentum:
$\Leftrightarrow J_\phi = 0, \qquad R = n$ M5^3 branes

Key observation

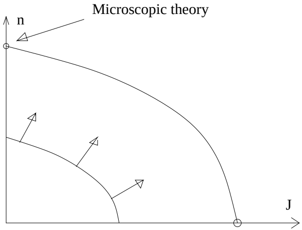
The solution becomes a self-dual null orbifold of $AdS_3 \times S^2$.

$$
\begin{aligned}
ds^2 &= l^2 \left(\frac{dr^2}{4r^2} - 2r dt dz \right) + \left(\frac{l}{2} \right)^2 d\Omega_2, \\
A &= -\frac{\sqrt{3}}{2} l \cos\theta d\phi,
\end{aligned} \qquad (1)
$$

where the identification $z \sim z + 2\pi \hat{L}_z$ breaks

$$
SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R \to U(1)_L \times SL(2,\mathbb{R})_R
$$

Key feature of the near-horizon limit



The supergravity approximation is valid as long as $R \gg 1$. So, we can take $n \to 0$.

Supersymmetry

$AdS_3 \times S^2$ is maximally supersymmetric in $N = 1$ five-dimensional minimal supergravity: it admits 8 real supercharges.

Three dimensional isometry supergroup is

$$SL(2, \mathbb{R})_L \times SU(1, 1|2)_R$$

At $J_\phi = 0$, the null orbifold preserves all supersymmetry

$$SL(2, \mathbb{R})_L \times SU(1, 1|2)_R \to U(1)_L \times SU(1, 1|2)_R$$

At $J_\phi > 0$, there is no global timelike or null Killing vector
$\Rightarrow$ Supersymmetry broken $\to U(1)_L \times (SL(2, \mathbb{R})_R \times U(1)_R)$.

Outline

1. Embedding in M-theory
   - Five-dimensional minimal supergravity and G2(2)
2. Magnetic Black String: Spinning M5 branes
   - Idea, The construction, Extremal Limit, Microscopics
3. Near Horizon Limit
   - Interpolating geometry $AdS_3 \times S^2$ to NHEK $\times S^1$
4. Microscopics
   - DLCQ MSW CFT, deformation, Kerr-CFT

MSW CFT Basics

Consider M-theory compactified on T^6 (or CY) in the regime

$$\frac{L_z}{l_{11}} \gg \frac{V_6}{l_{11}^6} \gg 1$$

Then, the worldvolume dynamics of intersecting M5 decouples from the bulk and is described by a $(0,4)$ CFT: the MSW CFT. In the regime of a large number of branes n

$$n^3 \gg \frac{V_6}{l_{11}^6}$$

the theory admits a supergravity holographic dual with $AdS_3 \times S^2$ asymptotics.

[Maldacena, Strominger, Witten, '97]

Dual Interpretation of the self-dual orbifold

Spacelike self dual orbifold of AdS_3 $\leftrightarrow$ left sector is a thermal density matrix, right sector is frozen. [Balasubramanian, de Boer, Sheikh-Jabbari, Simón, '09] [Balasubramanian, Parsons, Ross, '10]

Dual Interpretation of the self-dual orbifold

Spacelike self dual orbifold of AdS_3 $\leftrightarrow$ left sector is a thermal density matrix, right sector is frozen. [Balasubramanian, de Boer, Sheikh-Jabbari, Simón, '09] [Balasubramanian, Parsons, Ross, '10]

At any finite r the

CFT lives on a boosted cylinder

As $r \to \infty$, boost $\to \infty$. On the CFT this is Seiberg-Sen Discrete Light Cone Quantization (DLCQ) [Seiberg, Sen, '97].

Dual Interpretation of the self-dual orbifold

Spacelike self dual orbifold of $AdS_3 \leftrightarrow$ left sector is a thermal density matrix, right sector is frozen. [Balasubramanian, de Boer, Sheikh-Jabbari, Simón, '09] [Balasubramanian, Parsons, Ross, '10]

At any finite r the

CFT lives on a boosted cylinder

As $r \to \infty$, boost $\to \infty$. On the CFT this is Seiberg-Sen Discrete Light Cone Quantization (DLCQ) [Seiberg, Sen, '97].

Null self dual orbifold of $AdS_3 \leftrightarrow$ ground state of the DLCQ CFT. [de Boer, Sheikh-Jabbari, Simón] [Balasubramanian, Parsons, Ross] (Questionable: Closed Null Curves)

Kerr CFT as deformed DLCQ MSW CFT

Near horizon geometry of spinning $M5^3$-brane smoothly interpolates between susy null-orbifold of $AdS_3 \times S^2$ and NHEK $\times S^1$.

String theory on $AdS_3 \times S^2 \leftrightarrow$ MSW CFT.

SUSY null orbifold of $AdS_3 \times S^2 \leftrightarrow$ ground state of Discrete Light Cone Quantized (DLCQ) MSW CFT.

Turning on slightest amount of rotation ($J_\phi \sim \epsilon$), makes the orbifold spacelike, regulates the closed null curves

$$S = \frac{\pi^2}{3} c_L T_L = 2\pi J_\phi + O(\epsilon^3).$$

Kerr CFT as deformed DLCQ MSW CFT

Near horizon geometry of spinning $M5^3$-brane smoothly interpolates between susy null-orbifold of $AdS_3 \times S^2$ and NHEK $\times S^1$.

String theory on $AdS_3 \times S^2 \leftrightarrow$ MSW CFT.

SUSY null orbifold of $AdS_3 \times S^2 \leftrightarrow$ ground state of Discrete Light Cone Quantized (DLCQ) MSW CFT.

Turning on slightest amount of rotation ($J_\phi \sim \epsilon$), makes the orbifold spacelike, regulates the closed null curves

$$S = \frac{\pi^2}{3} c_L T_L = 2\pi J_\phi + O(\epsilon^3).$$

We can use AdS/CFT to study deviations away from the MSW CFT.

Operator deformations

The operators dual to the supergravity perturbations can be identified using the AdS/CFT dictionary. Two operators are turned on

	h_L	h_R	R-charge	spin $= h_R - h_L$
M_-	0	1	1	$+1$
M_+	2	1	1	-1

M_-: chiral primary of $SL(2,\mathbb{R})_L \times SU(1,1|2)_R$. M_+: J_- descendent of a chiral primary of $SL(2,\mathbb{R})_L \times SU(1,1|2)_R$.

- M_- related to spectral flow ($\phi \to \phi + 2\epsilon z$): preserves the conformal structure.
- M_+ is an irrelevant deformation: yet unidentified

Reformulation of the Kerr-CFT conjecture

If after a finite deformation by the operator M_+, the deformed theory is a CFT, then it is a microscopic model for Kerr-CFT

We conjecture more precisely

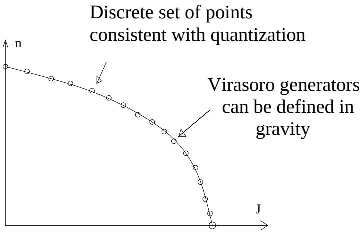
The deformed theory is a CFT with

$$c_D = 6R^3, \qquad T_D = \frac{J_\phi}{\pi R^3}, \qquad S_{BH} = \frac{\pi^2}{3} c_D T_D$$

The Kerr angular momentum is $J_\phi = 2R^3 L_z$.

The supergravity approximation is valid as long as $R \gg 1$.

Supporting arguments for a CFT at finite deformation



Our solution brings together many different ideas on Kerr/CFT in one coherent framework. We are now implementing this for the D1-D5-KK system.

Conclusions

1. Using hidden symmetries of five-dimensional minimal supergravity we have constructed the magnetic black string solution.
2. We have a proposal for a microscopic realization of Kerr-CFT.

$$\text{Susy Null orbifold of } AdS_3 \times S^2 \leftrightarrow NHEK \times S^1$$

3. Several Puzzles Remain: Self Dual Orbifold & AdS_2, boundary conditions, gravitational charges.

Thanks for your attention and for the invitation.